Mark Scheme 4755
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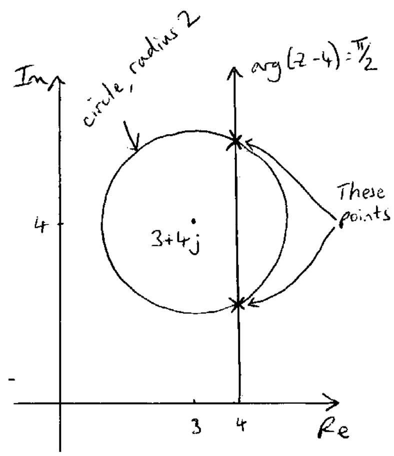
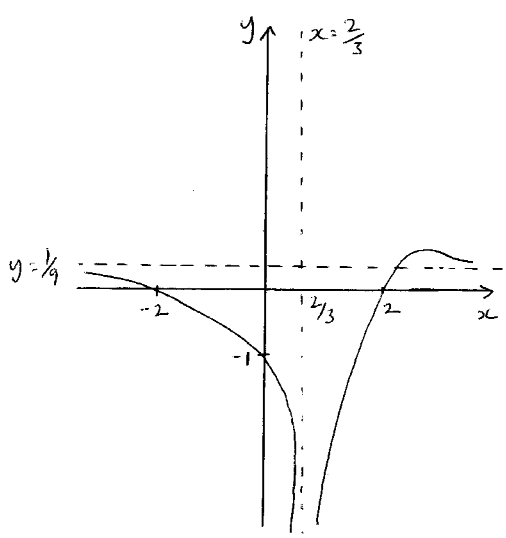
Section A			
1(i)	$\mathbf{A}^{-1} = \frac{1}{5}\begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix}$	M1 A1	Dividing by determinant
1(ii)	$\frac{1}{5}\begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 5 \\ -4 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 22 \\ -21 \end{pmatrix}$ $\Rightarrow x = \frac{22}{5},\ y = \frac{-21}{5}$	M1 A1(ft) , A1(ft) **[5]**	Pre-multiplying by their inverse Follow through use of their inverse No marks for solving without using inverse matrix
2	$4 - \mathrm{j},\ 4 + \mathrm{j}$ $\sqrt{17}\left(\cos 0.245 + \mathrm{j}\sin 0.245\right)$ $\sqrt{17}\left(\cos 0.245 - \mathrm{j}\sin 0.245\right)$	M1 A1 **[2]** M1 F1, F1 **[3]**	Use of quadratic formula Both roots correct Attempt to find modulus and argument One mark for each root Accept $(r,\ \theta)$ form Allow any correct arguments in radians or degrees, including negatives: 6.04, 14.0°, 346°. Accuracy at least 2s.f. S.C. F1 for consistent use of their incorrect modulus or argument (not both, F0)
3	$\begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow x = 3x - y,\ y = 2x$ $\Rightarrow y = 2x$	M1 A1 A1 **[3]**	M1 for $\begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ (allow if implied) $\begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} k \\ mk \end{pmatrix} = \begin{pmatrix} K \\ mK \end{pmatrix}$ can lead to full marks if correctly used. Lose second A1 if answer includes two lines
4(i) **4(ii)** **4(iii)**	$\alpha + \beta = 2,\ \alpha\beta = 4$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4 - 8 = -4$ Sum of roots = $2\alpha + 2\beta = 2(\alpha + \beta) = 4$	B1 M1A1 (ft) M1	Both Accept method involving calculation of roots Or substitution method, or method

	Product of roots = $2\alpha \times 2\beta = 4\alpha\beta = 16$ $x^2 - 4x + 16 = 0$	A1(ft) **[5]**	involving calculation of roots The = 0, or equivalent, is necessary for final A1

5(i)	Sketch of Argand diagram with: Point $3+4\mathrm{j}$. Circle, radius 2.	B1 B1 **[2]**	Circle must not touch either axis. B1 max if no labelling or scales. Award even if centre incorrect.
5(ii)	Half-line: Starting from (4, 0) Vertically upwards	B1 B1 **[2]**	
5(iii)	Points where line crosses circle clearly indicated. 	B1 **[1]**	Identifying 2 points where their line cuts the circle

Qu	Answer	Mark	Comment
Section A (continued)			
6	For $k=1$, $1^3=1$ and $\frac{1}{4}1^2(1+1)^2=1$, so true for $k=1$ Assume true for $n=k$ Next term is $(k+1)^3$ Add to both sides RHS = $\frac{1}{4}k^2(k+1)^2+(k+1)^3$ $=\frac{1}{4}(k+1)^2\left[k^2+4(k+1)\right]$ $=\frac{1}{4}(k+1)^2(k+2)^2$ $=\frac{1}{4}(k+1)^2((k+1)+1)^2$ But this is the given result with $(k+1)$ replacing *k*. Therefore if it is true for *k* it is true for $(k+1)$. Since it is true for $k=1$ it is true for *k* = 1, 2, 3, … .	B1 B1 B1 M1 M1 A1 E1 **[7]**	Assuming true for *k*, $(k+1)^{th}$ term - for alternative statement, give this mark if whole argument logically correct Add to both sides Factor of $(k+1)^2$ Allow alternative correct methods For fully convincing algebra leading to true for *k* $\Rightarrow$ true for *k* + 1 Accept 'Therefore true by induction' only if previous A1 awarded S.C. Give E1 if convincing explanation of induction following acknowledgement of earlier error
7	$3\sum r^2-3\sum r$ $=3\times\frac{1}{6}n(n+1)(2n+1)-3\times\frac{1}{2}n(n+1)$ $=\frac{1}{2}n(n+1)[(2n+1)-3]$ $=\frac{1}{2}n(n+1)(2n-2)$ $=n(n+1)(n-1)$	M1,A1 M1,A1 M1 A1 c.a.o. **[6]**	Separate sums Use of formulae Attempt to factorise, only if earlier M marks awarded Must be fully factorised
			Section A Total: 36

8(i)	$x=\frac{2}{3}$ and $y=\frac{1}{9}$	B1, B1 **[2]**	-1 if any others given. Accept min of 2s.f. accuracy
8(ii)	Large positive x, $y \to \frac{1}{9}^{+}$ (e.g. consider $x=100$) Large negative x, $y \to \frac{1}{9}^{-}$ (e.g. consider $x=-100$)	M1 A1 E1 **[3]**	Approaches horizontal asymptote, not inconsistent with their (i) Correct approaches Reasonable attempt to justify approaches
8(iii)	Curve $x=\frac{2}{3}$ shown with correct approaches $y=\frac{1}{9}$ shown with correct approaches (from below on left, above on right). (2, 0), (-2, 0) and (0, -1) shown 	B1(ft) B1(ft) B1(ft) B1 B1 **[5]**	1 for each branch, consistent with horizontal asymptote in (i) or (ii) Both x intercepts y intercept (give these marks if coordinates shown in workings, even if not shown on graph)
8(iv)	$-1=\frac{x^2-4}{(3x-2)^2} \Rightarrow -9x^2+12x-4=x^2-4$ $\Rightarrow 10x^2-12x=0$ $\Rightarrow 2x(5x-6)=0$ $\Rightarrow x=0$ or $x=\frac{6}{5}$ From sketch, $y \geq -1$ for $x \leq 0$ and $x \geq \frac{6}{5}$	M1 A1 B1 F1 **[4]**	Reasonable attempt at solving inequality Both values – give for seeing 0 and $\frac{6}{5}$, even if inequalities are wrong For $x \leq 0$ Lose only one mark if any strict inequalities given

9(i)	2 - j 2j	B1 B1 **[2]**	
9(iii)	$(x-2-\mathrm{j})(x-2+\mathrm{j})(x+2\mathrm{j})(x-2\mathrm{j})$ $=(x^2-4x+5)(x^2+4)$ $=x^4-4x^3+9x^2-16x+20$ So A = -4, B = 9, C = -16 and D = 20	M1, M1 A1,A1 A4 **[8]**	M1 for each attempted factor pair A1 for each quadratic - follow through sign errors Minus 1 each error – follow through sign errors only
OR	$-\mathrm{A}=\sum\alpha=4\Rightarrow\mathrm{A}=-4$ $\mathrm{B}=\sum\alpha\beta=9\Rightarrow\mathrm{B}=9$ $-\mathrm{C}=\sum\alpha\beta\gamma=16\Rightarrow\mathrm{C}=-16$ $\mathrm{D}=\sum\alpha\beta\gamma\delta=20\Rightarrow\mathrm{D}=20$	M1, A1 M1, A1 M1, A1 M1, A1 **[8]**	M1s for reasonable attempt to find sums S.C. If one sign incorrect, give total of A3 for A, B, C, D values If more than one sign incorrect, give total of A2 for A, B, C, D values
OR	Attempt to substitute two correct roots into $x^4+Ax^3+Bx^2+Cx+D=0$ Produce 2 correct equations in two unknowns A = -4, B = 9, C = -16, D = 20	M1 M1 A2 A4	One for each root One for each equation One mark for each correct. S.C. If one sign incorrect, give total of A3 for A, B, C, D values If more than one sign incorrect, give total of A2 for A, B, C, D values

10(i)	$\sum_{r=1}^{n}\frac{2}{r(r+1)(r+2)}=\sum_{r=1}^{n}\left[\frac{1}{r}-\frac{2}{(r+1)}+\frac{1}{(r+2)}\right]$	M1	Give if implied by later working
	$=\left(\frac{1}{1}-\frac{2}{2}+\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{2}{4}+\frac{1}{5}\right)+$ $........+\left(\frac{1}{n-1}-\frac{2}{n}+\frac{1}{n+1}\right)+\left(\frac{1}{n}-\frac{2}{n+1}+\frac{1}{n+2}\right)$	M1 A2	Writing out terms in full, at least three terms All terms correct. A1 for at least two correct
	$=\frac{1}{1}-\frac{2}{2}+\frac{1}{2}+\frac{1}{n+1}-\frac{2}{n+1}+\frac{1}{n+2}$ $=\frac{1}{2}-\frac{1}{n+1}+\frac{1}{n+2}$	M1 A3	Attempt at cancelling terms Correct terms retained (minus 1 each error)
	$=\frac{1}{2}-\frac{1}{(n+1)(n+2)}$	M1	Attempt at single fraction leading to given answer.
		[9]	
10(ii)	$\frac{1}{1\times 2\times 3}+\frac{1}{2\times 3\times 4}+\frac{1}{3\times 4\times 5}+...$ $=\frac{1}{2}\sum_{r=1}^{n}\frac{2}{r(r+1)(r+2)}=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{(n+1)(n+2)}\right)$	M1 M1	M1 relating to previous sum, M1 for recognising that $\frac{1}{(n+1)(n+2)}\to 0$ as $n\to\infty$ (could be implied)
	$\Rightarrow\frac{1}{1\times 2\times 3}+\frac{1}{2\times 3\times 4}+\frac{1}{3\times 4\times 5}+...=\frac{1}{4}$	A1	
		[3]	